glands are in some way connected with the absorption of nitrogenous matter from the putrid fluid in the cups, we are far from understanding the whole of the problem; for precisely similar *filament-protruding* glands are found on the seedling leaves of *D. sylvestris* and on the second year's leaves of *D. pilosus*; and as no "cups" are formed in either of these cases, the filaments cannot be connected with absorption of the products of decay. The only view which suggests itself is that the filaments absorb ammonia from the dew and rain. Recent researches have shown that certain leaves have the power of absorbing an appreciable quantity of ammonia; and this fact lends some probability to the view above advanced.

To recapitulate. Protoplasmic filaments are protruded from the leaf-glands of the teasel; and the only theory which seems at all capable of connecting the observed facts is the following:—That the glands on the teasel were aboriginally (*i. e.* in the ancestors of the Dipsacaceæ) mere resin-excreting organs; that the protoplasm which comes forth was originally a necessary concomitant of the secreted matters, but that, from coming in contact with nitrogenous fluids, it became gradually adapted to retain its vitality and to take on itself an absorptive function; and that this power, originally developed in relation to the ammonia in rain and dew, was further developed in relation to the decaying fluid accumulating within the connate leaves of the plant.

III. "On the Magnifying-power of the Half-prism as a means of obtaining great Dispersion, and on the General Theory of the 'Half-prism Spectroscope.'" By W. H. M. Christie, M.A., Fellow of Trinity College, Cambridge. Communicated by Dr. Huggins. Received January 25, 1877.

On account of the oblique incidence of the rays on the isosceles prism and the consequent diminution of the aperture of the collimator, a "half-prism," formed by dividing an isosceles prism by a plane perpendicular to the base, has frequently been employed for the commencement of the train of prisms, and also for the end, though apparently without due consideration of the effect of the "half-prism" on the dispersion of the other prisms preceding in the train. This is a matter of some importance; for it will be found that when the angles of incidence and emergence are unequal (as in the half-prism), the angular separation between two pencils of parallel rays is increased or diminished according as the angle of emergence is greater or less than the angle of incidence. In consequence of this the angle between the pencils corresponding to any two lines in the spectrum, *e. g.* the two D lines, will be increased by passing through a half-prism (independently of the effect of ordinary dispersion) if the perpendicular face be turned towards the slit. At the same time the

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angle between the pencils coming from the two edges of the slit will, for any monochromatic light, also be increased in nearly the same ratio, and thus the lines in any spectrum become broader as they are further separated by virtue of this property, and the purity of the spectrum remains unaltered. In fact the half-prism is from this point of view equivalent to a cylindrical magnifier, a property which may prove most useful in the construction of the spectroscope, leading to the substitution of three or, at the most, four half-prisms for the long train of fifteen or twenty isosceles prisms now used. For this purpose direct-vision prisms will be most suitable; but I propose to begin by considering the simpler case of a half-prism of flint. The expression "Magnifying-power" may be conveniently extended to the property in question, which, as will be seen presently, is strictly analogous to that of a lens; for it will be found that the same law holds with combinations of prisms as with other optical combinations, viz. the magnifying-power is the ratio of the breadth of the incident to the breadth of the emergent pencil*.

In consequence of the property of the half-prism stated above, it becomes necessary to distinguish between separation accompanied by increased breadth of the lines and separation in terms of this breadth. If the sense of the term "dispersion" had not been fixed by usage, it might very well be applied to the latter effect, retaining "separation" for the other; but it will undoubtedly be more in conformity with received ideas to define "dispersion" as the angular length of the spectrum, whilst the term "purity" may be used for the ratio of this angular length to the angular breadth of the spectrum lines (expressed in terms of the angular width of the slit). In the case of an isosceles prism, for which the magnifying-power is 1, the dispersion and purity are equal.

I will therefore distinguish the two positions in which a half-prism can be placed with respect to the incident rays as "Magnifying" and "Diminishing," according as the perpendicular face is turned towards the slit or away from it. It will be found that the first position will be suitable in cases where wide separation of the lines is required, and the second where great purity in the spectrum is of importance; but before entering on this point and on the further comparison with the ordinary combinations of isosceles prisms, it is desirable to find a convenient expression for the dispersion in different cases, confining this term to the effect of the variation in the refractive index.

* This law follows at once from the principle of conservation of energy, since the total energy of all the pencils which pass through the slit and fall (in a state of parallelism) on the prism is measured by the product of the breadth of the pencil into the angular width of the slit. If, therefore, by the refraction at any surface the breadth of the pencil is diminished, the angular breadth of the monochromatic image of the slit must be increased, the narrower pencils being spread over a larger angle, since the total energy remains constant. This is of course the ordinary law of brightness of optical images; a more formal proof from purely geometrical considerations is given in the text.

Simple Prism.

Let ϕ, ϕ' be the angles of incidence and refraction.

ψ, ψ' be the angles of emergence and refraction, ϕ and ψ being considered positive when they fall on the side of the normal away from the edge of the prism.

μ the refractive index.

α the refracting angle of the prism.

m the magnifying-power.

Δ the dispersion.

$\Pi=\frac{\Delta}{m}$ the purity.

D the deviation.

Then

$$\sin\phi=\mu\sin\phi', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\sin\psi=\mu\sin\psi', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\phi'+\psi'=\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$D=\phi+\psi-\alpha.$$

1. *Magnifying-power.*

Let $\delta\phi$=angle between two incident pencils.

$\delta\psi$=angle between the two corresponding emergent pencils.

Then

$$\text{Magnifying-power } m=-\frac{\delta\psi}{\delta\phi}.$$

Now (1) and (2) give, μ being constant,

$$\cot\phi\,\delta\phi=\cot\phi'\delta\phi', \text{ and } \cot\psi\,\delta\psi=\cot\psi'\delta\psi';$$

and from (3)

$$\delta\phi'=-\delta\psi',$$

$$m=\frac{\delta\psi}{\delta\phi}=-\frac{\delta\phi'}{\delta\phi}\cdot\frac{\delta\psi}{\delta\phi'};$$

$$\therefore m=\frac{\cot\phi}{\cot\phi'}\times\frac{\cot\psi'}{\cot\psi}=\frac{\cos\phi}{\cos\phi'}\cdot\frac{\cos\psi'}{\cos\psi}=\frac{\text{Breadth of incident pencil}}{\text{Breadth of emergent pencil}},$$

as will be seen at once by drawing perpendiculars to the incident, refracted, and emergent pencils. Also, if

$$m'=\frac{\cot\phi'}{\cot\phi}, \qquad m''=\frac{\cot\psi'}{\cot\psi},$$

we have

$$m=\frac{m''}{m'}.$$

We may call m', m'' the magnifying-powers of the first and second surfaces respectively, whence,

$$\text{Magnifying-power of half-prism}=\frac{\text{Magnifying-power of second surface}}{\text{Magnifying-power of first surface}}.$$

2. *Dispersion.*

Taking μ as variable,

$$\delta\phi = \frac{\delta\mu}{\mu}\tan\phi + \frac{\cot\phi'}{\cot\phi}\,\delta\phi', \qquad \delta\psi = \frac{\delta\mu}{\mu}\tan\psi + \frac{\cot\psi'}{\cot\psi}\,\delta\psi',$$

or

$$\delta\phi = \Delta' + m'\delta\phi', \qquad \delta\psi = \Delta'' + m''\delta\psi',$$

where

$$\Delta' = \frac{\delta\mu}{\mu}\tan\phi = \text{dispersion at first surface},$$

$$\Delta'' = \frac{\delta\mu}{\mu}\tan\psi = \text{dispersion at second surface};$$

also

$$\delta\phi' + \delta\psi' = 0, \text{ and } \delta D = \delta\phi + \delta\psi.$$

Eliminating $\delta\phi'$ and $\delta\psi'$, we have

$$m''\delta\phi + m'\delta\psi = m''\Delta' + m'\Delta'',$$

with the further relations,

$$\left.\begin{aligned} \delta\phi - m'\delta\phi' &= \Delta', \\ \delta\psi - m''\delta\psi' &= \Delta''. \end{aligned}\right\}$$

3. *Purity.*

Putting $\Pi' = \frac{\Delta'}{m'} = \text{purity at first surface},$

$$\Pi'' = \frac{\Delta''}{m''} = \text{purity at second surface},$$

we have

$$\Pi' = \frac{\delta\mu}{\mu}\tan\phi', \text{ and } \Pi'' = \frac{\delta\mu}{\mu}\tan\psi';$$

$$\therefore\ m''\delta\phi + m'\delta\psi = m'm''(\Pi' + \Pi'').$$

Before proceeding further we must assume some relation between $\delta\phi$, $\delta\psi$, $\delta\phi'$, $\delta\psi'$ which will give a fourth equation; and here three cases may be considered.

i. $\delta\phi = 0$, or the collimator fixed relatively to the prism. This gives the dispersion and purity of the spectrum as seen by the eye, whatever be the form of spectroscope used.

ii. $\delta\phi' = -\delta\psi' = 0$, or the collimator and telescope movable relatively to the prism in such a manner that the pencils for different parts of the spectrum all pass through the prism in the same direction. This is the condition in the ordinary form of spectroscope with automatic adjustment to minimum deviation.

iii. $\delta D = 0$, or the collimator and telescope relatively fixed, whilst the prism is turned so as to bring different parts of the spectrum into the field of view. This can only be done when the angles of incidence and emergence are unequal, as in the half-prism spectroscope.

Taking these three cases respectively, we have

i. $\delta\phi=0$; $\therefore \Delta=m\Delta'+\Delta''$, $\Pi=m'(\Pi'+\Pi'')$.

ii. $\delta\phi'=-\delta\psi'=0$; $\therefore \delta\phi=\Delta'=m'\Pi'$, $\delta\psi=\Delta''=m''\Pi''$, and $\delta D=\Delta'+\Delta''=m'\Pi'+m''\Pi''$.

iii. $\delta D=0$; $\therefore \delta\phi=-\delta\psi=$angle through which the prism is turned $=\theta$.

Then $\theta=\dfrac{m''\Delta'+m'\Delta''}{m''-m'}=\dfrac{m\Delta'+\Delta''}{m-1}=\dfrac{\Delta}{m-1}=\dfrac{m}{m-1}\Pi$.

In the second and third cases the scales of the dispersion and purity are variable, and are given by (i) for the part of the spectrum under examination on substituting the corresponding values of ϕ, ϕ', ψ, ψ' in m', m'', Δ', Δ''.

The three forms of simple prism which we have specially to consider are :—I. Half-prism magnifying ; II. Half-prism diminishing ; III. Isosceles prism.

I. *Half-prism magnifying.*

1. Magnifying-power .. $m_{,}=\dfrac{m''}{\mu}=\dfrac{\cos\psi'}{\cos\psi}$, since ϕ and $\phi'=0$ and $m'=\mu$.

2. Dispersion ..... $\Delta_{,}=\Delta''=\dfrac{\delta\mu}{\mu}\tan\psi=m_{,}\delta\mu\tan\psi'$, since $\Delta=0$.

3. Purity ..... $\Pi_{,}=\delta\mu\tan\psi'$.

II. *Half-prism diminishing.*

Keeping ϕ and ϕ' as the angles at the perpendicular face, which is now that of emergence, and taking $m_{,,}$, $\Delta_{,,}$, and $\Pi_{,,}$ as the values of the magnifying-power, dispersion, and purity in this case, we have :—

1. Magnifying-power $m_{,,}=\dfrac{1}{m_{,}}=\dfrac{\cos\psi}{\cos\psi'}$, m' and m'' being interchanged.

2. Dispersion ..... $\Delta_{,,}=\dfrac{1}{m_{,}}\Delta''+\Delta'=\dfrac{\Delta_{,}}{m_{,}}=\dfrac{1}{m_{,}}\cdot\dfrac{\delta\mu}{\mu}\tan\psi=\delta\mu\tan\psi'$,

since $\Delta'=0$.

3. Purity ..... $\Pi_{,,}=\dfrac{\dfrac{\Delta_{,}}{m_{,}}}{\dfrac{1}{m_{,}}}=\Delta_{,}=m_{,}\Pi_{,}=\dfrac{\delta\mu}{\mu}\tan\psi=m_{,}\delta\mu\tan\psi'$.

III. *Isosceles prism.*

1. Magnifying-power .. $m_{,,,}=1$, since $\phi=\psi$, $\phi'=\psi'$, and $m'=m''$.

2. Dispersion ..... $\Delta_{,,,}=\Delta'+\Delta''=2\dfrac{\delta\mu}{\mu}\tan\psi=2m'\,\delta\mu\tan\psi'$.

3. Purity ..... $\Pi_{,,,}=\Delta_{,,,}=2\Delta_{,}$.

These results might also have been obtained by considering the prism as made up of two half-prisms separated by a thin plate of air. The dispersion of the first half is $\frac{\Delta_1}{m_1}$, and this is magnified m_1 times by the second, which also adds its own dispersion Δ_1, thus giving $\frac{\Delta_1}{m_1} \cdot m_1 + \Delta_1 = 2\Delta_1$. This will be seen at once to be equivalent to the algebraic process employed.

The above results may be summed up as follows:—

1. The purity of a half-prism magnifying is equal to the dispersion of a half-prism diminishing, and is the product of the dispersive power of the glass and the tangent of the angle of the prism.

2. The dispersion of a half-prism magnifying is equal to the purity of a half-prism diminishing, and is m times the purity of a half-prism magnifying.

3. The purity and dispersion of an isosceles prism are equal, and are each twice the dispersion of a half-prism magnifying.

In future when the letters m, Δ, and Π are used without suffixes, they must be understood as denoting the magnifying-power, dispersion, and purity of a half-prism magnifying.

Confining our attention for the present to the ordinary form of a spectroscope in which $\delta\phi' = -\delta\psi' = 0$, the most important question to be considered is the irrationality of dispersion, which in fact fixes the limit to the refracting angle of the prism, unless the spectroscope is to be used only for one part of the spectrum.

The formula for the dispersion is chiefly useful as giving the irrationality, which is seen to increase with the dispersion, but in a more rapid ratio. In fact if we put Δ_A and Δ_H for the dispersions corresponding to the same small value of $\frac{\delta\mu}{\mu}$ for the Fraunhofer lines A and H, *i. e.* the values of the scale of the spectrum at those two points, we have

$$\log \frac{\Delta_H}{\Delta_A} = \log \tan \psi_H - \log \tan \psi_A = \frac{\sec^2 \psi}{\tan \psi} (\psi_H - \psi_A) \text{ approximately}$$

$$= \frac{2(\psi_H - \psi_A)}{\sin 2\psi} = \frac{\delta\mu}{\mu} \cdot \sec^2 \psi.$$

In practice, however, it will be more convenient to express the ratio in terms of the magnifying-powers for A and H; thus

$$\frac{\Delta_H}{\Delta_A} = \frac{m_H}{m_A} \cdot \frac{\delta\mu_H}{\delta\mu_A},$$

if the prism be adjusted to minimum deviation in each case.

In the following Table the angles of emergence, the dispersion, and the magnifying-power are given for a series of half-prisms of very dense

flint, the refractive indices for A and H being $\mu_A = 1{\cdot}700$ and $\mu_H = 1{\cdot}770$. The refracting angle of the half-prism $= \psi'$. In this case

$$\Delta = \frac{1}{25}\tan\psi \text{ and } \frac{\Delta}{m} = \frac{1}{14}\tan\psi'.$$

Half-prism of Flint.

Dispersion and Magnifying-power.

ψ'.	ψ_H.	ψ_A.	Δ.	$m_H = \frac{\cos\psi'}{\cos\psi_H}$.	$m_A = \frac{\cos\psi'}{\cos\psi_A}$.
°	° ′	° ′	° ′		
22	41 35	39 36	1 59	1·24	1·20
24	46 5	43 47	2 18	1·32	1·27
26	50 50	48 7	2 43	1·42	1·35
28	56 7	52 52	3 15	1·57	1·45
30	62 15	58 13	4 2	1·86	1·64
32	69 44	64 17	5 27	2·45	1·95
34	81 40	71 48	9 52	5·72	2·66
34½	..	74 20	..	..	3·05
35	..	77 10	..	..	3·69
35½	..	80 45	..	..	5·12
36	..	87 43	..	..	20·20

Light reflected at emergence.

ψ'.	Polarized		Total.
	in plane of incidence.	in perpendicular plane.	
°	per cent.	per cent.	per cent.
30	H 28·5	0·0	14·3
	A 22·4	0·0	11·2
32	H 39·1	2·5	20·8
	A 28·9	0·5	14·7
34	H 67·1	27·7	47·4
	A 40·5	4·8	22·7
34½	A 45·9	8·3	27·1
35	A 52·2	13·3	32·8
35½	A 62·6	24·8	43·7
36	A 94·4	71·6	83·0

The above numbers express the percentage of light lost by reflexion, computed by Fresnel's formulæ for H and A respectively.

These Tables will enable us readily to compare prisms of different angles in the case of this particular quality of glass, which has been selected as giving large dispersion with moderate absorption. Let us take two examples for comparison with direct-vision prisms.

1. Isosceles prism of 60°.

$\Delta = 8° 4'$. Loss of light for H = 25 per cent.
for A = 20 per cent.

Irrationality of dispersion

$2\delta\psi_A = 2m_A\,\delta\mu \tan\psi' = \cdot 135$ or $7° 44'$.

$2\delta\psi_H = 2m_H\,\delta\mu \tan\psi' = \cdot 153$ or $8° 46'$.

2. Isosceles prism of 64°.

$2\Delta = 10° 54'$. Loss of light for H = 34 per cent.
for A = 25 per cent.

Irrationality of dispersion

$2\delta\psi_A = 2m_A\,\delta\mu \tan\psi' = \cdot 174$ or $9° 58'$.

$2\delta\psi_H = 2m_H\,\delta\mu \tan\psi' = \cdot 218$ or $12° 30'$.

In computing the loss of light after passing through several surfaces, the two polarized beams must of course be taken separately and afterwards combined.

The quantities given under Irrationality of Dispersion represent what would be the length of the spectrum if the scale were the same throughout as at A and H respectively. This is independent of irrationality in the refractive index of glass as referred to wave-lengths.

If we had taken an ordinary flint, the above numbers would be considerably modified. Thus taking $\mu_A = 1{\cdot}6200$, and $\mu_H = 1{\cdot}6666$, which correspond to a moderately dense flint, Fraunhofer's No. 30, specific gravity 3·7, we should have

For a prism of 60°,

$$\psi_A = 54° 6',\ \psi_H = 56° 25',\ \text{dispersion} = 4° 38'.$$

Compound Prism.

Let β be the refracting angle, μ' the refractive index of the prism of crown glass which is cemented to the half-prism of flint,
χ, χ' the angles of emergence and refraction at the outer surface of the crown,

the other angles being denoted as before, though the relation between ψ and ψ' will now be

$$\mu' \sin\psi = \mu \sin\psi'.$$

Two cases here present themselves according as β is greater or less than ψ. The former gives the direct-vision prism, and the latter the compound prism commonly used in large spectroscopes.

In the first case we have $\psi + \chi' = \beta$, in the second $\psi - \chi' = \beta$; but this can be reduced to the first case by considering χ' as negative. In conformity with this we shall consider the angles of refraction as positive when they fall on the side of the normal towards the edge of the prism (whether flint or crown) in which the refraction takes place, and the angles of incidence or emergence as positive when they fall on the side from the edge.

Thus the two forms of compound prism may be considered together; but there is this important distinction, that the refraction at the outer surface of the crown corrects irrationality of dispersion in the direct-vision prism, since the angle of refraction is greater for the red rays, whilst in the ordinary compound prism the irrationality is greatly increased. At the same time the refraction at the outer surface of the crown slightly decreases the dispersion for the whole spectrum in the direct-vision prism, and slightly increases it in the other form; but this effect is produced in the latter case by spreading out the violet rays, and in the former by bringing them closer together, the dispersion for the red rays being about the same for the same angles of refraction in the two forms, as will be seen more clearly when an actual case is taken. Thus the apparent superiority of the ordinary compound prism turns out to be illusory, being gained by inequality in the scale; and this is aggravated by corresponding loss of light for the violet rays, as the deviation at *both* refractions is greater than for the red rays. On the other hand, both the scale and the loss of light are equalized throughout the spectrum in a properly constructed direct-vision prism, which is therefore, independently of its practical convenience, the best form of compound prism for general use.

Compound Prism.—We have here

$$\sin\phi=\mu\sin\phi';\quad \mu'\sin\psi=\mu\sin\psi';\quad \sin\chi=\mu'\sin\chi';$$
$$\phi'+\psi'=\alpha;\quad \psi+\chi'=\beta;\quad \mathrm{D}=\phi-\chi-\alpha+\beta;$$

the deviation D being considered positive when the pencil is bent from the refracting edge of the flint prism.

1. *Magnifying-power.* μ, μ' being constant,

$$m=\frac{\delta\chi}{\delta\phi}=\frac{\delta\phi'}{\delta\phi}\cdot\frac{\delta\psi'}{\delta\phi'}\cdot\frac{\delta\psi}{\delta\psi'}\cdot\frac{\delta\chi'}{\delta\psi}\cdot\frac{\delta\chi}{\delta\chi'}=\frac{\cot\phi}{\cot\phi'}\times-1\times\frac{\cot\psi'}{\cot\psi}\times-1\times\frac{\cot\chi'}{\cot\chi}$$
$$=\frac{\cos\phi}{\cos\phi'}\cdot\frac{\cos\psi'}{\cos\psi}\cdot\frac{\cos\chi'}{\cos\chi}=\frac{m''m'''}{m'},$$

where

$$m'=\frac{\cot\phi'}{\cot\phi}=\mu\frac{\cos\phi'}{\cos\phi};\quad m''=\frac{\cot\psi'}{\cot\psi}=\frac{\mu}{\mu'}\frac{\cos\psi'}{\cos\psi};\quad m'''=\frac{\cot\chi'}{\cot\chi}=\mu'\frac{\cos\chi'}{\cos\chi}.$$

2. *Dispersion.*—Taking logarithms and differentiating, μ and μ' being now variable,

$$\delta\phi=\frac{\delta\mu}{\mu}\tan\phi+\frac{\cot\phi'}{\cot\phi}\delta\phi';\quad \delta\psi=\left(\frac{\delta\mu}{\mu}-\frac{\delta\mu'}{\mu'}\right)\tan\psi+\frac{\cot\psi'}{\cot\psi}\cdot\delta\psi';$$
$$\delta\chi=\frac{\delta\mu'}{\mu'}\tan\chi+\frac{\cot\chi'}{\cot\chi}\delta\chi',$$

or

$$\delta\phi=\Delta'+m'\delta\phi';\quad \delta\psi=\Delta''+m''\delta\psi';\quad \delta\chi=\Delta'''+m'''\delta\chi';$$

where

$$\Delta'=\frac{\delta\mu}{\mu}\tan\phi,\quad \Delta''=\left(\frac{\delta\mu}{\mu}-\frac{\delta\mu'}{\mu'}\right)\tan\psi,\quad \Delta'''=\frac{\delta\mu'}{\mu'}\tan\chi,$$

Δ', Δ'', Δ''' being the dispersion at the first, second, and third surfaces respectively; also

$$0=\delta\phi'+\delta\psi',\quad 0=\delta\psi+\delta\chi',\quad \text{and } \delta D=\delta\phi-\delta\chi.$$

Eliminating $\delta\phi'$ and $\delta\psi'$,

$$m''\delta\phi+m'\delta\psi=m''\Delta'+m'\Delta'';$$

and eliminating $\delta\psi$ and $\delta\chi'$, we have

$$m''m'''\delta\phi-m'\delta\chi=m''m'''\Delta'+m'm'''\Delta''-m'\Delta''',$$

with the three further relations,

$$\begin{aligned}\delta\phi-m'\delta\phi' &=\Delta',\\ \delta\psi-m''\delta\psi' &=\Delta'',\\ \delta\chi-m'''\delta\chi' &=\Delta'''.\end{aligned}$$

3. *Purity.*—Putting Π', Π'', Π''' for the purity of the first, second, and third surfaces respectively,

$$\Pi'=\frac{\Delta'}{m'},\quad \Pi''=\frac{\Delta''}{m''},\quad \Pi'''=\frac{\Delta'''}{m'''},$$

and we have

$$\Pi'=\frac{\delta\mu}{\mu}\tan\phi',\quad \Pi''=\left(\frac{\delta\mu}{\mu}-\frac{\delta\mu'}{\mu'}\right)\tan\psi',\quad \Pi'''=\frac{\delta\mu'}{\mu'}\tan\chi',$$

$$m''m'''\delta\phi-m'\delta\chi=m'm''m'''(\Pi'+\Pi'')-m'm'''\Pi'''.$$

Taking the three cases as before, we have:—

i. $\delta\phi=0$, $\therefore \Delta=m\Delta'+m'''\Delta''-\Delta'''=m''m'''(\Pi'+\Pi'')-m'''\Pi'''$

$$=m''m'''\left\{\frac{\delta\mu}{\mu}\tan\phi'+\frac{\delta\mu}{\mu}-\frac{\delta\mu'}{\mu'}\tan\psi'\right\}-m'''\frac{\delta\mu'}{\mu'}\tan\chi';$$

$$\Pi=m'\left(\Pi'+\Pi''-\frac{\Pi'''}{m''}\right).$$

ii. $\delta\phi'=-\delta\psi'=0$ and $\delta\psi=-\delta\chi'=0$,

$$\therefore \delta\phi=\Delta'=m'\Pi',\ \delta\psi=\Delta''=m''\Pi'',\ \delta\chi=\Delta'''=m'''\Pi''',$$

and $$\delta D=\Delta'-\Delta'''=m'\Pi'-m'''\Pi'''.$$

iii. $\delta D=0$, $\therefore \delta\phi=\delta\chi=$ angle through which prism is turned $=\theta$.

Then

$$\theta=\frac{m\Delta'+m'''\Delta''-\Delta'''}{m-1}=\frac{\Delta}{m-1}=\frac{m}{m-1}\Pi.$$

Taking the three forms of compound prism, we have:—

I. *Half-prism magnifying.*

1. Magnifying-power $m_1=\dfrac{m''m'''}{\mu}=\dfrac{\cos\psi'}{\cos\psi}\cdot\dfrac{\cos\chi'}{\cos\chi}$.

2. Dispersion $\ldots\ldots \Delta_1=m'''\Delta''-\Delta'''=m_1\mu\left(\dfrac{\delta\mu}{\mu}-\dfrac{\delta\mu'}{\mu'}\right)\tan\psi'-\dfrac{\delta\mu'}{\mu'}\tan\chi'$.

3. Purity $\ldots\ldots\ldots \Pi_1=\mu\left(\Pi''-\dfrac{\Pi'''}{m''}\right)=\dfrac{\Delta_1}{m_1}$

II. *Half-prism diminishing.*

1. Magnifying-power . . $m_{II} = \frac{1}{m_I}$.

2. Dispersion . . . . . $\Delta_{II} = \frac{\Delta_I}{m_I}$.

3. Purity . . . . . $\Pi_{II} = m_I \Pi_I = \Delta_I$.

III. *Isosceles prism.*

1. Magnifying-power . . $m_{III} = 1$.

2. Dispersion . . . . . $\Delta_{III} = 2\Delta_I$.

3. Purity . . . . . $\Pi_{III} = 2\Delta_I$.

In order to fix the ideas, let us take the same flint, as in the case of the simple prism, and for the crown glass $\mu_A = 1{\cdot}510$, $\mu_H = 1{\cdot}530$. Then

$$\frac{\delta\mu}{\mu} = \frac{1}{25},\ \frac{\delta\mu'}{\mu'} = \frac{1}{75},\ \text{and}\ \frac{\delta\mu}{\mu} - \frac{\delta\mu'}{\mu} = \frac{2}{75},\ \text{and}\ \mu = \frac{7}{4}\ \text{very nearly};$$

$$\therefore \Delta = -\frac{1}{150}\left\{7m \tan\psi' - 2\tan\chi\right\}.$$

The first term increases with ψ', or the angle of the half-prism of flint, which should therefore be as large as practicable, consistently with good definition and moderate loss of light: it also increases with m; but in that case the second term increases too in so far as m depends on χ. It will therefore be desirable to find the rate of variation of m with χ, supposing ψ and ψ' to remain constant. Since

$$m = \frac{\cos\chi'}{\cos\chi} \cdot \frac{\cos\psi'}{\cos\psi},$$

we have, taking logarithms of both sides and differentiating,

$$\frac{\delta m}{m} = \tan\chi\,\delta\chi - \tan\chi'\,\delta\chi';$$

whence, since

$$\cos\chi\,\delta\chi = \mu'\cos\chi'\,\delta\chi',\ \text{and}\ \frac{\tan\chi'}{\tan\chi} = \frac{1}{\mu'}\frac{\cos\chi}{\cos\chi'},$$

$$\delta m = m\tan\chi\left\{1 - \frac{1}{\mu'^2}\frac{\cos^2\chi}{\cos^2\chi'}\right\}\delta\chi$$

$$= m\tan\chi \cdot \frac{\mu'^2 - 1}{\mu'^2 - \sin^2\chi} \cdot \delta\chi,$$

which increases rapidly with χ, and

$$\delta \cdot \tan\chi = \frac{\delta\chi}{\cos^2\chi}.$$

Taking as an example $\chi = 60°$ and $m = 3$, we have

$$\delta m = \frac{5}{2}\sqrt{3} \cdot \delta\chi = 4{\cdot}3\delta\chi \text{ and } \delta \cdot \tan\chi = 4\delta\chi.$$

The following Tables are arranged in the same form as those for the simple prism, taking

$$\mu_A = 1{\cdot}700,\ \mu'_A = 1{\cdot}510,\ \text{and therefore}\ \frac{\mu_A}{\mu'_A} = 1{\cdot}130,$$

$$\mu_H = 1{\cdot}770,\ \mu'_H = 1{\cdot}530,\qquad \frac{\mu_H}{\mu'_H} = 1{\cdot}160.$$

Crown. Dispersion and Magnifying-power.

χ'.	χ_H.	χ_A.	$\frac{m_H'''}{\mu_H} = \frac{\cos\chi'}{\cos\chi_H}$.	$\frac{m_A'''}{\mu'_A} = \frac{\cos\chi'}{\cos\chi_A}$.
°	° ′	° ′		
0	0 0	0 0	1·000	1·000
4	6 8	6 3	1·003	1·003
10	15 26	15 14	1·022	1·021
14	21 44	21 26	1·044	1·035
18	28 13	27 49	1·080	1·076
22	35 1	34 29	1·132	1·125
26	42 9	41 25	1·210	1·199
28	45 55	45 9	1·270	1·253
30	49 54	49 2	1·345	1·320
31	52 0	51 3	1·393	1·364
32	54 11	53 8	1·450	1·413
33	56 26	55 20	1·518	1·474
34	58 47	57 35	1·600	1·547
35	61 21	60 0	1·708	1·638
36	64 2	62 33	1·847	1·755
37	67 2	65 21	2·047	1·915
38	70 18	68 21	2·338	2·136
39	74 20	71 51	2·880	2·495
40	79 32	76 9	4·210	3·210
41		82 7		5·510

Light Reflected.

χ'.	Polarized		Total.
	in plane of incidence.	in perpendicular plane.	
°	per cent.	per cent.	per cent.
0	4·6	4·6	4·6
10	5·1	4·2	4·7
18	6·3	3·2	4·8
26	9·4	1·4	5·4
30	12·3	0·5	6·4
34	H 17·6	0·0	8·8
	A 16·0	0·0	8·0
36	H 22·7	0·9	11·8
	A 20·5	0·6	10·6
38	H 31·6	4·4	18·0
	A 27·7	3·0	15·4
39	H 40·7	9·4	25·1
	A 33·6	6·0	19·8
40	H 53·4	21·7	37·6
	A 43·2	12·8	28·0
41	A 61·7	32·6	47·2

The dispersion is simply the difference between corresponding quantities in the two columns χ_H and χ_A.

The magnifying-power (given in the last two columns for H and A) in this and following Tables is to be understood as that of a half-prism having χ' (or ψ') for its refracting angle.

Flint to Crown.

Dispersion and Magnifying-power.

ψ'.	ψ_H.	ψ_A.	$\psi_H - \psi_A$.	$\frac{\mu'}{\mu} m_H'' = \frac{\cos\psi'}{\cos\psi_H}$.	$\frac{\mu'}{\mu} m_A'' = \frac{\cos\psi'}{\cos\psi_A}$.
45°	55° 6′	53° 2′	2° 4′	1·24	1·18
47	58 1	55 44	2 17	1·29	1·21
49	61 6	58 31	2 35	1·36	1·26
51	64 21	61 26	2 55	1·45	1·32
53	67 52	64 29	3 23	1·60	1·39
55	71 50	67 46	4 4	1·84	1·52
56	74 5	69 32	4 33	2·04	1·60
57	76 35	71 23	5 12	2·35	1·71
57½	78 3	72 22	5 41	2·60	1·77
58	79 38	73 24	6 14	2·95	1·86
58½	81 30	74 28	7 2	3·53	1·95
59	83 53	75 37	8 16	4·82	2·07
59½	88 10	76 49	11 21	15·86	2·23

Light Reflected.

ψ'.	Polarized		Total.
	in plane of incidence.	in perpendicular plane.	
	per cent.	per cent.	per cent.
45°	3·2	0·1	1·6
49	5·0	0·6	2·8
53	H 8·9	2·5	5·7
	A 5·0	1·1	3·1
55	H 13·1	5·1	9·1
	A 6·9	2·1	4·5
56	H 16·4	7·6	12·0
	A 8·3	3·0	5·7
57	H 21·4	11·5	16·5
	A 10·0	4·1	7·1
58	H 29·9	18·9	24·4
	A 12·5	5·9	9·2
59	H 48·6	37·5	43·1
	A 16·1	8·6	12·4
59½	H 80·4	74·6	74·5
	A 18·6	10·7	14·7

For the purpose of comparing the direct-vision and ordinary compound

prism, we may take as a particular case $\alpha = 56°$, and $\beta = 106° 32'$ for the first, and $37° 38'$ for the second, so that the extreme angle of incidence or emergence at the crown will be the same in both forms, viz. $65° 21'$.

We thus have

Direct-vision Half-prism.		*Compound Half-prism.*	
$\alpha = 56° 0'$	$\beta = 106° 32'$	$\alpha = 56° 0'$	$\beta = 37° 38'$
$\chi_A = 65\ 21$		$\chi_A = 52\ 56$	
$\chi_H = 55\ 11$		$\chi_H = 65\ 21$	
Dispersion $= 10\ 10$		Dispersion $= 12\ 25$	

Deviation

for A $= -14\ 49$,	for A $= +34\ 34$,
for H $= -\ 4\ 39$,	for H $= +46\ 59$,

the deviation being considered positive when it is in the same direction as that of the flint prism.

Magnifying-power.

$m_A = 3{\cdot}07$	$m_A = 2{\cdot}26$
$m_H = 3{\cdot}04$	$m_H = 3{\cdot}96$

Irrationality of Dispersion.

$$\delta\chi_A = -\frac{1}{150}(31{\cdot}8 - 4{\cdot}4) \qquad \delta\chi_{A_1} = -\frac{1}{150}(23{\cdot}4 + 2{\cdot}6)$$

$$= -{\cdot}183 \text{ or } -10° 26' \qquad = -{\cdot}173 \text{ or } -9° 55'$$

$$\delta\chi_H = -\frac{1}{150}(31{\cdot}5 - 2{\cdot}9) \qquad \delta\chi_H = -\frac{1}{150}(41{\cdot}0 + 4{\cdot}4)$$

$$= -{\cdot}191 \text{ or } -10° 57' \qquad = -{\cdot}303 \text{ or } -17° 22'.$$

Light lost by Reflexion.

	Polarized		Total.	Polarized		Total.
	in plane of incidence.	in perpendicular plane.		in plane of incidence.	in perpendicular plane.	
	per cent.	per cent.	per cent.	per cent.	per cent.	per cent.
For A	36·5	11·3	23·9	27·1	10·4	18·8
For H	34·7	15·3	25·0	44·1	15·6	29·9

The advantage of the direct-vision prism is thus clearly shown, whether as compared with a compound prism of the other form, or with two simple prisms of the same kind of flint and giving the same dispersion.

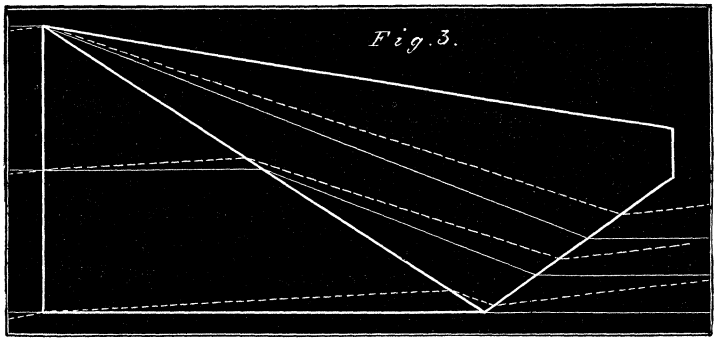
In fact the direct-vision half-prism here considered gives nearly the same dispersion as a simple prism of 64°, with less loss of light and without the great irrationality which makes the dispersion in the other case merely illusory.

It is to be remarked that there still remains the irrationality in the variation of the refractive index for glass, which may, however, be par-

tially corrected in the direct-vision prism by increasing the angle of the crown. The prism would, however, no longer be direct-vision, even for the ultra-violet, the deviation being negative and considerable in amount for the visible rays.

So far the case has been considered in which the pencils for different parts of the spectrum all pass in the same direction through the prism (parallel to the base), and the collimator and telescope are moved; but it is also possible to keep the collimator and telescope fixed and to turn the prism merely. In the former plan, which is that usually adopted, the limit to the angle of the prism is fixed by the condition that the angle of incidence or emergence for the violet at the oblique face must not be too large, or, in other words, that the angle of refraction must not be very near the critical angle for the violet, which is smaller than that for the red; and consequently great dispersion is accompanied with corresponding irrationality, except in the case of the direct-vision prism. If, however, the deviation be kept the same for all parts of the spectrum—which can be done by turning the prism (provided it be not isosceles)—this objection is got rid of, and at the same time the construction of the spectroscope is made much more simple. The refracting angle of a half-prism, whether simple or compound, can thus be increased, so as to give the most suitable dispersive power for the middle of the dispersion spectrum (about F) when the pencil is incident perpendicularly on the first face. By turning the prism successively in one direction or the other, so as to increase the angle of refraction for the red or diminish it for the violet, the different parts of the spectrum can be made to emerge successively in the same direction, and the dispersive and magnifying-powers, as well as the loss of light, will be sensibly constant throughout the spectrum, the dispersion at the first face being extremely small. Of course, if the whole spectrum be included in the field of view there will be the usual inequality of scale; but such an extent of field can only be used for viewing a spectrum with small dispersion; and for purposes of measurement, the part examined must be brought to the middle of the field. It may be remarked that the dispersion may be reduced to about one half by turning the half-prism till the angles of incidence and emergence become equal, when it acts as an isosceles prism at minimum deviation. This property is useful for the examination of faint objects, the decrease in dispersion being further accompanied by an increase in the transmitted light. The spectrum, however, would in this case be thrown some 3° out of the centre of the field, and a motion either of the viewing-telescope or of the eyepiece would be necessary.

In fig. 3 is shown the course of a pencil of red rays (full lines) and of violet rays (dotted lines) within a direct-vision half-prism as it is turned about a centre in the manner just described. The rays, whether red or violet, emerge parallel to their original direction, the diagram representing the course relative to the prism as it is turned.



Taking the case of a single half-prism with fixed collimator and spectroscope, we may increase the refracting angle (or the angle of refraction for mean rays), and thus obtain much larger dispersion. In comparing the three forms of prism (simple, direct-vision, and ordinary compound), the irrationality was the most important consideration; but this does not here enter into the question. The direct-vision prism, however, has still the advantage, as compared with the compound prism, of giving a more convenient form of spectroscope, whilst, as we can now use very deep angles for the flint, the dispersion at the exterior surface of the crown hardly affects the resulting dispersion.

As compared with a simple half-prism of flint, there is the advantage of being able to make ψ' (or the angle of the flint) nearly twice as large, with a corresponding increase in the purity; for it will be remarked that although the magnifying-power and dispersion may be increased almost indefinitely, there is a limit to the purity given by making $\psi' =$ the critical angle; and consequently there will be great advantage in a combination of flint and crown surfaces which increases the critical angle without materially diminishing the dispersive power.

In illustration of this the following Table may be interesting. It is calculated for a specimen of very dense flint glass (spec. grav. 5·0) of remarkable purity, which Mr. Hilger has procured, and with which he has made to my instructions several experimental direct-vision half-prisms. The refractive indices of this glass for A and H are $\mu_A = 1{\cdot}7595$ and $\mu_H = 1{\cdot}8411$.

$\psi' = a$.	ψ_A.	$\frac{\mu'}{\mu} m_A''$.	$D'' = -D'''$.	χ'_A.	χ_A.	β.	m.	Π.	Δ.
°	° ′		° ′	° ′	° ′	° ′		° ′	°
54	70 37	1·77	16 37	27 25	44 2	98 2	2·19	4 43	10·3
55	72 47	1·94	17 47	28 41	46 28	101 28	2·43	4 49	11·7
56	75 10	2·19	19 10	30 8	49 18	105 18	2·89	4 55	14·2
56½	76 29	2·36	19 59	30 57	50 56	107 26	3·21	5 1	16·1
57	77 55	2·61	20 55	31 49	52 44	109 44	3·78	5 7	19·3
57½	79 33	2·97	22 3	32 46	54 49	112 19	4·56	5 14	23·8
58	81 27	3·56	23 27	33 53	57 20	115 20	5·50	5 20	29·4
58½	83 49	4·84	25 19	35 13	60 32	119 2	8·35	5 26	45·5
59	88 0	14·76	29 0	37 20	66 20	125 20	29·20	5 32	162·6

This Table gives the angles of refraction and emergence and the deviations at the two refractions (D″, D‴), and also the magnifying-power, purity, and dispersion for various direct-vision half-prisms, α being the angle of the flint and β the angle of the crown.

The light lost by reflexion at the refraction from flint to crown is as follows:—

Light lost by Reflexion.

$\psi' = \alpha$.	Polarized		Total.
	in plane of incidence.	in perpendicular plane.	
°	per cent.	per cent.	per cent.
54	12·1	4·2	8·2
56	19·1	9·2	14·2
56½	22·0	11·5	16·8
57	25·4	14·5	20·0
57½	30·4	18·9	24·7
58	37·7	25·7	31·7
58½	49·2	37·5	43·4
59	79·1	72·9	76·0

As the amount of light lost in different combinations of prisms requires careful consideration, I have by the help of the preceding Tables calculated the loss in passing through trains of half-prisms having respectively angles of 57° for the flint, with 110° for the crown, and 58½° for the flint, with 120° for the crown. These particular forms of half-prism (which I will call A and B) have been used in two experimental spectroscopes to be described shortly, and I have therefore selected them as examples.

Light lost by Reflexion.

	Polarized.		Total.	Dispersion.
	in plane of incidence.	in perpendicular plane.		
	per cent.	per cent.	per cent.	°
One half-prism A . .	40·9	21·2	31·0	19·3
Two half-prisms A .	65·1	37·9	51·5	73·0
Three „ „ . .	79·4	51·1	65·2	276·0
Four „ „ . .	87·8	61·4	74·6	1043·0
One half-prism B . .	61·7	42·5	52·1	45·5
Two half-prisms B .	85·3	66·9	76·1	380·0

It will be seen that for equal dispersion the loss of light is slightly greater for a train of half-prisms B; but this is fully compensated for by the diminished absorption, as fewer prisms will be required.

In considering the most advantageous combination of prisms in a spectroscope, it will be necessary to bear in mind the different purposes for which the instrument is used. The requirements in a spectroscope may thus be divided into the following heads:—

(1) Wide separation of the lines, with little loss of light and moderate purity in the spectrum.

(2) Great purity in the spectrum.

(3) Wide separation of the lines accompanied by considerable purity.

In (2) and (3) loss of light is of secondary importance. For measuring the displacement of lines in the spectra of stars and work of a similar character (1) must be specially considered; and here the width of the slit is determined by the size of the star's image rather than by the angle subtended at the object-glass of the collimator. With prisms of fair size sufficient purity will usually be obtained for measurement of the strong lines without limiting the slit further; the most important point is large aperture in the equatoreal, as the amount of light collected varies with the aperture. This will readily be seen by considering that the angle subtended by the diffraction image of a star at the optical centre of the object-glass (or mirror) varies inversely as the aperture; hence for prisms of a given size the purity of the spectrum will be the same for all apertures, if the focal length of the collimator correspond.

For a bright-line spectrum, such as that of the solar prominences, which is seen projected on the continuous spectrum of the sky and of which the visibility depends on the contrast with the background and not on the absolute brightness, (2) is of the greatest importance; whilst special attention must be paid to (3) in the case of the sun.

We may compare three arrangements of prisms:—(1) Half-prisms magnifying; (2) Half-prisms diminishing; (3) Whole prisms; but in doing this we must take account of the width of the slit and the breadth of the pencil as determining the purity of the spectrum and its brightness. At the same time, as the size of the prisms practically fixes the limit to the power of the spectroscope, the quantity of glass used in the different cases should also be borne in mind. This latter consideration is important, because in a train of half-prisms magnifying the breadth of the pencil is diminished in the ratio $1 : m$ with each half-prism (the height remaining unaltered), and the reverse takes place with a train of half-prisms diminishing. Thus in both cases the half-prisms may be made narrower and narrower in geometrical progression, starting with that end which begins with a perpendicular face. The difference between the two cases is, that in the first case the train (counting from the slit) begins with the broadest prism, and in the second case with the narrowest, so that with the latter arrangement we have a narrower pencil and consequently less light. In the case of a train of whole prisms the pencil is narrower than for the half-prisms magnifying, in the ratio of $1 : m$. The thickness of the glass through which the rays pass is another important consideration; and from this point of view the half-prism train has a great advantage. The breadth

of the spectrum, as compared with the height of the slit, should also be taken into account, for this affects the brightness directly. Hitherto advantage has not been taken of the circumstance that the height of the slit is practically unlimited (except in the case of stars), and that therefore it is unnecessary to magnify the spectrum in this direction. Thus the advantage of the train of "half-prisms magnifying" is obvious, since the breadth of the spectrum is left unaltered. In spectroscopes as ordinarily constructed, the breadth of the spectrum is magnified at the same time as the separation of the lines, and thus a high magnifying-power is accompanied by a great decrease in the brightness, the latter varying inversely as the square of the power. By using a cylindrical eyepiece, however, this objection may be removed; and there would appear to be no practical difficulty in this plan, since little or no enlargement of the breadth of the spectrum is required. For bright-line spectra generally a Ramsden eyepiece with cylindrical lenses would answer the purpose well, the height of the slit being sufficient to give a convenient breadth to the spectrum without any amplification in this direction; but for viewing the solar prominences the magnifying-power should be the same in both directions; and thus with the train of "half-prisms diminishing," which, as will be seen, gives great purity in the spectrum with a corresponding decrease in the breadth of the image of the slit, the magnifying-power of the eyepiece must be high in the direction of the breadth of the slit, and low in the perpendicular direction. An eyepiece formed with a cylindrical concave or Barlow lens (within the principal focus of the viewing-telescope) and an ordinary convex field-lens in combination with a convex cylindrical eye-lens, would produce this effect, the axes of both cylindrical lenses being parallel to the slit.

The effect of such an eyepiece may easily be investigated.

Let f_1, f_2 be the focal lengths of the concave and convex cylindrical lenses, f the focal length of the convex field-lens, and F the focal length of the viewing-telescope; then, if M, M′ be the magnifying-powers parallel and perpendicular to the slit,

$$M=-\frac{F}{f}.$$

For the rays in the perpendicular plane, suppose the concave cylindrical lens to form an enlarged image on the field-lens, which will therefore produce no effect, this image being at the principal focus of the cylindrical eye-lens.

Then if v be the distance from the concave lens of the focus after refraction, and m' the enlargement of the image, f and v are both negative, and

$$\frac{v}{v-f}=m'\ ;\quad \therefore\ v=\frac{m'}{m'-1}f,\quad v-f=\frac{1}{m'-1}f.$$

Also

$$\frac{1}{v}-\frac{1}{v-f}=\frac{1}{f_1}\ ;\quad \therefore\ f_1=\frac{-m'}{(m'-1)^2}f,$$

and

$$M' = \frac{m'\ F}{f_2}.$$

If we make $f_2 = -f_1$ so as to equalize the two cylindrical lenses, we have $M' = -(m'-1)^2 \cdot \frac{F}{f}$. Thus if

$$M' = 9\frac{F}{f}, \quad m' = 4, \text{ and } f_1 = -f_2 = \tfrac{4}{9}f;$$

$$M' = 25\frac{F}{f}, \quad m' = 6, \text{ and } f_1 = -f_2 = \tfrac{6}{25}f.$$

Taking 1 inch as the equivalent focal length of an ordinary eyepiece, we should get in the perpendicular direction magnifying-powers 9 and 25 times as great with cylindrical lenses of $\frac{4}{9}$ and $\frac{6}{25}$ inch respectively. There would probably be no difficulty in applying such lenses.

We shall see, however, that one or more small half-prisms placed between the eye-lens and the eye would answer the same purpose, with the advantage of increasing the dispersion; and this arrangement appears to be decidedly superior to a cylindrical eyepiece.

We may now find expressions for the angular breadth of the lines, the dispersion and the purity of the spectrum in the different cases, as well as the quantity of glass used for the train of prisms and its total thickness.

Let

p = number of whole prisms.
n = number of half-prisms.
m = magnifying-power of each.
2Δ = dispersion of each whole prism.
Q = quantity of glass in the smallest half-prism of the train.
t = thickness of glass in the smallest half-prism of the train.
b_1, b_2, and b_3 = breadth of slit in the three cases.
$1-c$ = coefficient of absorption per unit of thickness.
$1-k_1$ and $1-k_2$ = proportion lost by reflexion in passing through one half-prism of the light polarized in and perpendicular to plane of incidence.
$1-k_1$ and $1-k_2$ = similar quantities for a whole prism.

1. *Half-prisms magnifying.*

By what precedes it will be clear that each half-prism in the train will magnify the angular separation of two pencils falling on it, whether coming from the two edges of the slit, or corresponding to two lines in the spectrum (such as the two D lines). Thus the breadths of the two D lines will be magnified in the same ratio as their separation, due to the dispersion of the preceding prisms, and to this will be added the dispersion proper (as distinct from the magnifying-power) of the half-prism itself.

Thus, there being n half-prisms in the train,

Breadth of spectrum-lines .. $=m^n . b_1$.

Dispersion ..... $=(m^{n-1}+m^{n-2}+\ldots+m^2+m+1)\Delta$

$$=\frac{m^n-1}{m-1}\cdot\Delta$$

Purity of spectrum ..... $=\frac{m^n-1}{m-1}\cdot\frac{1}{m^n}\cdot\frac{\Delta}{b_1}$.

Quantity of glass ..... $=(m^{2(n-1)}+m^{2(n-2)}+\ldots+m^4+m^2+1)Q$

$$=\frac{m^{2n}-1}{m^2-1}\cdot Q.$$

Thickness of glass..... $=(m^{n-1}+m^{n-2}+\ldots+m^2+m+1)t$

$$=\frac{m^n-1}{m-1}\cdot t.$$

Total brightness of spectrum.. $=b_1\cdot\frac{1}{2}(k_1{}^n+k_2{}^n)c^{\frac{m^n-1}{m-1}\cdot t}$.

2. *Half-prisms diminishing.*

Breadth of spectrum-lines .. $=\frac{1}{m^n}\cdot b_2$.

Dispersion ..... $=\left(\frac{1}{m^{n-1}}+\frac{1}{m^{n-2}}+\ldots+\frac{1}{m^2}+\frac{1}{m}+1\right)\frac{\Delta}{m}$

$$=\frac{m^n-1}{m-1}\cdot\frac{1}{m^n}\cdot\Delta.$$

Purity of spectrum ..... $=\frac{m^n-1}{m-1}\cdot\frac{\Delta}{b_2}$.

Quantity and thickness of glass the same as for "half-prisms magnifying."

Total brightness of spectrum.. $=\frac{b_2}{m^n}\cdot\frac{1}{2}(k_1^n+k_2^n)c^{\frac{m^n-1}{m-1}\cdot t}$.

3. *Whole prisms.*

Breadth of spectrum-lines .. $=b_3$.

Dispersion ..... $=2p\,.\,\Delta$.

Purity of spectrum ..... $=2p\,.\,\frac{\Delta}{b_3}$.

Quantity of glass ..... $=2p\,.\,m^{2(n-1)}\,.\,Q$.

Thickness ..... $=2p\,.\,m^{n-1}\,.\,t$.

Total brightness of spectrum.. $=\frac{b_3}{m}\,.\,\frac{1}{2}\,(k_1{}^p+k_2{}^p)c^{2pm^{n-1}t}$.

k_1 and k_2 are nearly equal to $k_1{}^2$ and $k_2{}^2$ respectively.

4. *A train of* n *half-prisms magnifying followed by a train of* n *half-prisms diminishing.*

Breadth of spectrum lines .. $=1$.

$$\text{Dispersion}=\text{purity} \ldots\ldots = (m^{n-1}+m^{n-2}+\ldots+m+1).\Delta.\frac{1}{m^n} + \left(\frac{1}{m^{n-1}}+\frac{1}{m^{n-2}}+\ldots+\frac{1}{m}+1\right).\frac{\Delta}{m}$$

$$= 2\frac{m^n-1}{m-1}.\frac{\Delta}{m^n}.$$

$$\text{Total brightness of spectrum} \ldots = b_1.\tfrac{1}{2}(k_1^{2n}+k_2^{2n})c^{2\frac{mn-1}{m-1}.t}.$$

5. *A train of* n *half-prisms diminishing followed by a train of* n *half-prisms magnifying.*

$$\text{Breadth of spectrum-lines} \ldots = 1$$

$$\text{Dispersion}=\text{purity} \ldots\ldots = \left(\frac{1}{m^{n-1}}+\frac{1}{m^{n-2}}+\ldots+\frac{1}{m}+1\right)m^n\Delta + (m^{n-1}+\ldots+m+1)\Delta$$

$$= 2\frac{m^n-1}{m-1}.\Delta.$$

$$\text{Total brightness of spectrum} \ldots = \frac{b_2}{m^n}.\tfrac{1}{2}(k_1^{2n}+k_2^{2n})c^{2\frac{m^n-1}{m-1}t}.$$

The second train of half-prisms in (4) and (5) may be placed between the eye-lens and the eye, and will then conveniently replace the cylindrical magnifier. The prisms may in that case be very small; their dispersion, however, will not be magnified by the eyepiece, and they will therefore add but little to the dispersion or purity.

In fact if M be the magnifying-power of the viewing-telescope, the dispersion would be $(M+1)\frac{m^n-1}{m-1}.\frac{\Delta}{m^n}$ and $(M+1)\frac{m^n-1}{m-1}.\Delta$, and the purity $\frac{M+1}{M}.\frac{m^n-1}{m-1}.\frac{\Delta}{m^n}$ and $\frac{M+1}{M}.\frac{m^n-1}{m-1}.\Delta$ for (4) and (5) respectively.

This is on the supposition that the second train has the same magnifying-power as the first.

Now for the same brightness of spectrum, neglecting loss of light by reflexion and absorption, we must have $b_2=m^n b_1$ and $b_3=mb_1$, the breadth of the slit being increased to compensate for the decrease in the breadth of the incident pencil. Where this is not practicable (as for star-spectra) the train of "half-prisms magnifying" has the advantage of greater brightness; and, as it gives at the same time wide separation of the lines, it best fulfils the condition (1).

On the other hand, for viewing the solar prominences, loss of light is of little or no consequence, the important point being to have a strong contrast between the monochromatic image of the slit (filled with the light of the prominence) and the dispersed image due to the light of the sky. Thus the breadth of the slit does not enter into the question, and great purity in the spectrum is the chief desideratum. A train of "half-prisms diminishing" is evidently by far the best arrangement; but for

convenience in delineating the prominences, a cylindrical eyepiece or a train of half-prisms magnifying (between the eye-lens and the eye) must be used.

Such a combination is also suitable for spectrum-analysis, where a bright-line spectrum has to be distinguished from a continuous spectrum forming the background. Thus with this arrangement the sodium lines in the flame of an ordinary paraffin candle are seen with great brilliancy.

In general, however, a combination of wide separation, purity, and brightness is required; and here we must take the values for b_2 and b_3 given above in terms of b_1.

(1) and (2) give the same purity $\frac{m^n-1}{m-1}\cdot\frac{1}{m^n}\cdot\frac{\Delta}{b_1}$

(3) gives the purity ..... $2p\,\frac{1}{m_1}\cdot\frac{\Delta}{b_4}$.

If we take the same number of half-prisms in the three cases, $2p=n$; and (3) is superior in this respect to (1) and (2), the purity being proportional to $1+1+1+\ldots$ to n terms and to $1+\frac{1}{m}+\frac{1}{m^n}+\ldots$ to n terms in the two cases respectively.

But as the practical question is how to get the best result out of a given quantity of glass, we should divide the purity by the quantity of glass, and thus we get the ratio for what may be called the efficiency,

$$E_1=\frac{m+1}{m^n+1}\cdot\frac{1}{m^n}\cdot\frac{\Delta}{b_1} \text{ for (1) and (2),}$$

and

$$E_3=\frac{1}{m^{2n-1}}\cdot\frac{\Delta}{b_1} \text{ for (3).}$$

Hence

$$\frac{E_1}{E_3}=\frac{m^n+m^{n-1}}{m^n+1}=1+\frac{m^{n-1}-1}{m^n+1},$$

which shows the advantage of the half-prism train, m being greater than 1.

On the other hand, there is a small additional loss of light at the perpendicular faces in the half-prism train amounting to 6 or 7 per cent. for each half-prism; but as not more than three, or at the most four, are ever likely to be required, even for the sun, this loss is hardly sensible.

As regards the *thickness* of glass, the ratio of purity to thickness of glass is the same in all three cases, viz. :—

$$\frac{1}{m^n}\cdot\frac{\Delta}{b}\,\frac{1}{t}.$$

We may consider one other point, viz. the effect of error in the surfaces and of unequal density in the glass, though the question is a very obscure one. An error in the surface being equivalent to a slight deviation in the incident ray, its effect will vary as the magnifying-power; and thus with the train of half-prisms magnifying, the errors of the half-prisms are magnified in proportion to their distance from the end of the

train in the same ratio as the dispersions; and if there is no tendency to a repetition of similar errors throughout the train, these must be combined according to the ordinary law of errors of observation, and thus we should get probable error from whole train $=\sqrt{\frac{m^{2n}-1}{m^n-1}}\times$ probable error from one half-prism.

The train of "half-prisms diminishing" would give the same result, since a magnifying-power of m^n must be used to give the same separation. In the case of the whole prisms, a magnifying-power of $\frac{m^n-1}{m-1}\cdot\frac{1}{p}$ is required, and therefore probable error from whole train $=\frac{m^n-1}{m-1}\cdot\frac{1}{2\sqrt{p}}$ × probable error from a whole prism. Thus, considering that a higher magnifying-power must be applied where few prisms are used, the case of a long train of prisms would be equivalent to the arithmetical mean of a large number of fallible surfaces; but, independently of the fact that after a very moderate number the gain in accuracy of definition is very slight, practical experience seems to show that, owing probably to systematic errors in the surfaces or the glass, an increase in the number of prisms is accompanied by a corresponding loss of definition, and that consequently the law of errors does not apply. Hitherto a long train of prisms has been considered necessary, partly because it has been the only means of obtaining purity, *i. e.* dispersion as distinct from mere separation, and partly because the use of high magnifying-powers involved the weakening of the light by increase in the apparent breadth of the spectrum. As regards the effect of air-bubbles and other defects in the glass in stopping out light, it falls under the head of absorption, and constitutes a very serious objection to the long train of prisms.

Probably, from want of proper data, the effect of absorption in spectroscopes would appear to have been generally overlooked; and in many cases where compound prisms are employed the brightness of the spectrum is far inferior to what would have been obtained with much smaller prisms. Even with a train of simple prisms the loss by absorption is very great, and to this is superadded the loss by reflexion at the numerous surfaces, so that the disadvantages of a long train are sufficiently obvious. To obtain some information on this point, I have made several series of measures of the brightness of the light transmitted through various thicknesses of different kinds of glass, using a polarizing photometer and a chromatometer (on the principle of Prof. Clerk-Maxwell's colour-box); and though the observations I have made are not so extensive as I could wish for the elucidation of this interesting physical question, they are amply sufficient for the immediate object in view, showing conclusively the wasteful loss of light in the ordinary construction of the spectroscope. There would seem to be great differences in the absorptive power of different specimens of glass, depending on the manufacture rather than on the density, though all the specimens of very dense flint which I have

examined absorb the blue and violet rays strongly, whilst crown glass absorbs the red and green. Notwithstanding its yellow colour, very dense flint (having a specific gravity of 5·0), when well made, actually transmits considerably more light than the specimens of crown I have met with. Thus I found that for a thickness of four inches the proportion of the incident light transmitted in the case of flint (sp. gr. 5·0) and crown was as follows, allowance being made for the loss by reflexion at the surfaces:—

	Red. per cent.	Green. per cent.	Blue. per cent.
Flint	56	49	5
Crown	40	39	21

From these results it would appear that a thickness of 4 inches of flint or 3 inches of crown would absorb half of the incident light, the glass being in each case well made.

The absorption in the case of a train of prisms can easily be calculated as follows:—

Let e^{-ft}, e^{-ct} be the coefficient of light transmitted through thicknesses t, t' of flint and crown.

h, k the height and breadth of the half-prism.

κ the abscissa for any ray measured from the edge of the flint prism.

$t=q\kappa$, $t'=q'(k-\kappa)$, where $q=n\tan\alpha$, $q'=\dfrac{n\sin\beta}{\cos\alpha\cos(\beta-\psi)}$, n being the number of half-prisms.

Putting B= Brightness of transmitted pencil, which is supposed to be large enough to fill the whole of the prism, we have

$$B=Ch\int_0^k e^{-ft}e^{-ct'}d\kappa=Ch\int_0^k e^{-(fq-cq')\kappa-ct'k}\,d\kappa$$

$$=\frac{Ch}{-fq+cq'}\cdot\{e^{-fqk}-e^{-cq'k}\},\ C \text{ being a constant.}$$

For evaluating this expression the following Table of the values of the negative exponential will be found convenient:—

·95	$e^{-0\cdot051}$	·75	$e^{-0\cdot288}$	·55	$e^{-0\cdot598}$	·35	$e^{-1\cdot050}$	·15	$e^{-1\cdot897}$
·90	$e^{-0\cdot105}$	·70	$e^{-0\cdot357}$	·50	$e^{-0\cdot693}$	·30	$e^{-1\cdot204}$	·10	$e^{-2\cdot303}$
·85	$e^{-0\cdot162}$	·65	$e^{-0\cdot431}$	·45	$e^{-0\cdot798}$	·25	$e^{-1\cdot386}$	·05	$e^{-2\cdot996}$
·80	$e^{-0\cdot223}$	·60	$e^{-0\cdot511}$	·40	$e^{-0\cdot916}$	·20	$e^{-1\cdot609}$	·02	$e^{-3\cdot912}$
								·01	$e^{-4\cdot605}$

1. *Simple Prism.*

$$q'=0 \qquad B=\frac{Ch}{fq}\cdot\{1-e^{-fqk}\};$$

thus B increases with k, rapidly at first, and then more and more slowly.

Taking a prism of 60°, and putting $f=0\cdot173$, which would give an absorption of 50 per cent. for 4 inches, we have the following Table for the brightness $\frac{1}{n}\{1-e^{-fqk}\}$ (omitting the constant factor $\dfrac{Ch}{2f\tan\alpha}$):—

Slant side	½ in.	1 in.	2 in.	3 in.	4 in.	6 in.	∞
No. of Prisms.	Brightness.						
1	·084	·16	·29	·40	·50	·65	1·00
2	·080	·145	·25	·325	·375	·44	0·50
4	·073	·125	·19	·22	·235		0·25
8	·063	·095	·118				0·125
16	·048	·059	·062				0·063

These figures show how little is gained by increasing the *breadth* of the prisms in a long train; in fact the

$$\text{Brightness} = \frac{\text{Absorption through the bases of all the prisms}}{\text{Number of prisms}};$$

and this absorption, as it approaches its limiting value 1, increases very slowly with the thickness of glass when that exceeds 6 or 8 inches. The *height* of the prisms may, however, be increased with advantage up to the limit determined by the condition that the height of the emergent pencil (after passing through the eyepiece) shall not exceed the diameter of the pupil of the eye. Thus with a magnifying-power of 10 on the viewing-telescope it is useless to make the prisms more than 2 inches in height.

The coefficient $\frac{Ch}{2f\tan a}$ shows the variation of the brightness with the absorptive power of the glass and the angle of the prism.

The above formula has been obtained on the supposition that the whole of the prism is utilized right up to the refracting edge, which is assumed to have no sensible thickness. In practice, however, this is rarely the case, and the most valuable part of the prism is lost, from the difficulty of working a surface true up to the edge. In a long train of prisms this is a matter of some importance, where light is of consequence; and there is in such a train the further objection, that the pencil falling on the viewing-telescope is in fact rendered excentrical, and its effective breadth diminished, by the greater absorption in passing through the thick parts of the prisms.

2. *Compound Prisms.*

Here, as the breadth of the prism is increased, there will be a point where the increased absorption makes up for the increased brightness of the incident pencil; and when this is the case

$$\frac{d\mathrm{B}}{dk} = 0, \text{ or } \frac{Ch}{fq - cq'}\{fqe^{-fqk} - cq'e^{-cq'k}\} = 0,$$

$$\therefore fqe^{-fqk} = cq'e^{-cq'k},$$

whence

$$k = \frac{1}{-fq + cq'} \cdot \log \frac{cq'}{fq};$$

suppose $fq = xcq'$,

$$\therefore\ k = \frac{1}{(x-1)cq'} \cdot \log x,$$

which gives the limiting breadth k.

The condition $e^{-cq'k} = xe^{-fqk}$ may be expressed in words thus:—When the brightness is a maximum, the light transmitted through the thickest parts of the train of crown prisms must be x times that through the thickest parts of the train of flint prisms, x being the thickness of crown (expressed in terms of the total thickness corresponding to 1 inch breadth of prism) which gives the same absorption as the total thickness of flint corresponding to 1 inch breadth. The thickness is in each case that traversed by the ray.

When $x=1$ or $fq=cq'$, the formula becomes indeterminate; the absorption is then the same for all rays of the pencil, and

$$B = Chke^{-fqk} = Chke^{-cq'k}.$$

Putting $\frac{dB}{dk} = 0$ as before, we have

$$k = \frac{1}{fq} \text{ or } \frac{1}{cq'};$$

or, when the brightness is a maximum, the breadth of the prisms must be such that the whole absorption is $\frac{1}{e}$, the absorption in passing through the crown being supposed equal to that in the flint. This will be approximately true in the case of the direct-vision prism, for which therefore the thickness of glass traversed (flint and crown combined) should not exceed 5·8 inches, if we take $f = \cdot 173$, as found by experiment for a very good specimen of flint. With less pure glass, such as is frequently used, this limiting thickness may be much smaller; in one piece of very dense flint which I examined it would be only 1·5 inch; but the absorption of the crown being here considerably less than that of the flint, the formula just given would no longer apply. Further, for the blue end of the spectrum f would be much larger, being, in fact, about ·78 for the yellow flint, whilst c is about ·34 for the crown, so that the total thickness for photographic work should certainly not exceed 3 inches, the limit being fixed by the absorption of the crown.

These numerical results, however, are given more as illustrating the principles on which spectroscopes should be constructed than as general rules applicable in all cases; for there is such great diversity in glass that, by a judicious selection, it is quite possible that a considerably greater thickness might prove to be admissible for the photographic rays. Further, the observations on which the values of the coefficients of absorption are based are liable to a small uncertainty on account of the difficulty of allowing for want of perfect polish in the two surfaces of the plate. They may be taken, however, as useful approximations to the truth.

From what precedes it will be seen that loss of light is so inseparably associated with dispersion, whether by making it necessary to use a narrower pencil or in other ways which are not so easily reduced to law, that there is theoretically but little to choose between different arrangements of prisms, and in particular between a long train of prisms and a short train of half-prisms; but practically the advantage seems to be altogether with the short train, which is less complicated and more convenient. The difficulties of the adjustment of a long train of prisms are so great that in practice the efficiency secured appears to fall very far short of the theoretical limit, and thus so-called powerful spectroscopes are a constant source of trouble and disappointment to those who use them. Even if once properly adjusted it is practically impossible to keep them so, and the consequence is that the sodium lines, for example, are seen hardly any better with eighteen or twenty prisms than with four or five.

The general conclusions arrived at with regard to the best combination of prisms may be summed up as follows:—

1. The direct-vision prism (as ordinarily used) possesses great advantages over other forms, simple or compound, especially in its correction of the irrationality of dispersion.

2. This irrationality can, however, be corrected in any form of "half-prism" by keeping the collimator and telescope relatively fixed, whilst the prism only is turned to bring successive parts of the spectrum into the field of view, and thus an angle approaching closely to the critical angle may be advantageously used.

3. A train composed of three or at most four "half-prisms (direct-vision) magnifying" is the best arrangement for giving wide separation of the lines in the solar spectrum, whilst one or, in some cases, two half-prisms similarly placed offer special advantages in the case of star-spectra. A well-worked slit will be required for examination of the finer lines in the sun's spectrum, and it would be well to use a concave or "Barlow" lens in the collimator so as to increase its effective focal length.

4. A train of two or three "half-prisms (direct-vision) diminishing," in combination with an eyepiece train of "half-prisms magnifying," is peculiarly adapted for the delineation of the solar prominences, and generally for the examination of bright-line spectra with a continuous background.

Mr. Hilger, who has entered with the greatest zeal into the practical application of these principles, and to whose skill the success of the idea is largely due, has made a direct-vision spectroscope for the Royal Observatory on the following plan (see fig. 1), which has been approved by the Astronomer Royal, and which embodies the above conclusions. The train consists of three half-prisms direct-vision for the red rays, each having an angle of 57° 0′ for the flint (sp. gr. 5·0) and 110° for the crown, and giving a dispersion of 20° with a magnifying-power of 3·8. The course of the rays is sufficiently indicated in the diagram, from which it

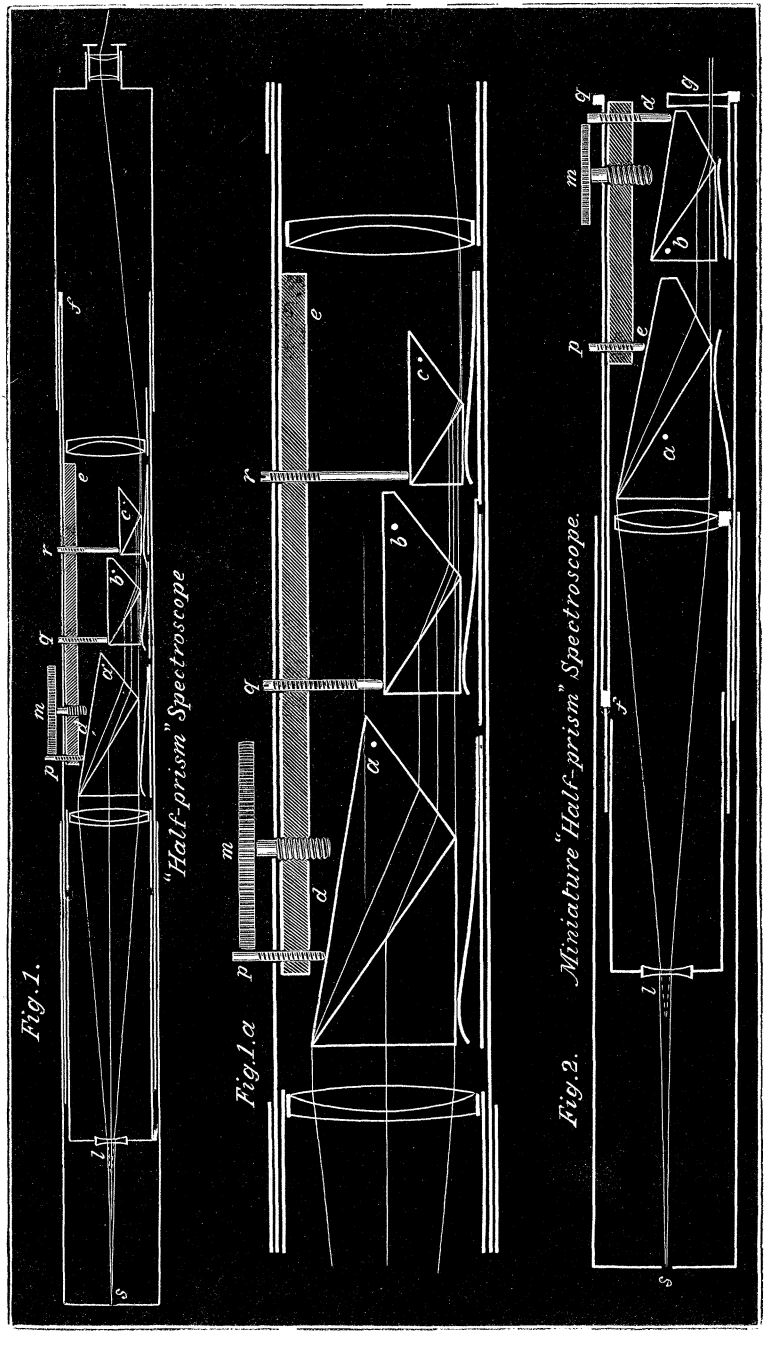


Fig. 1.
"Half-prism" Spectroscope
Fig. 1.a
Fig. 2. Miniature "Half-prism" Spectroscope.

will be seen how the breadth of the pencil decreases, the rays being thrown to one side, as it were, by the refraction of the prisms. The prisms turn about centres a, b, c, so placed that the rays for different parts of the spectrum, after emerging from one prism, always fall on the same part of the face of the next prism, each being turned through the same angle. There is no difficulty in calculating the positions of these points in any particular case from the condition that $\delta\phi = \delta\chi$ as the prism is turned, and that the lateral displacement of the middle ray in one prism must be equal and opposite to the lateral motion of the perpendicular face of the next prism. For the sake of convenience in mounting (the centres being in the glass itself) the second and third prisms have been made larger than is absolutely necessary; and besides this a large portion of the crown might be cut off as far as optical requirements are concerned. The prisms are each turned through the same angle by the lever $d\,e\,f$, having its fulcrum at f and moved by the micrometer-screw m.

This lever carries the three adjustable screws p, q, r, so placed as to give the same angular motion to each prism, which is kept in contact with the corresponding screw by a spring on the opposite side. Thus a simple motion of the lever by means of the micrometer-screw brings different parts of the spectrum into the field; and this automatic motion will not be deranged in the least by withdrawing either one or two of the prisms, which are carried in short adapter tubes to admit of easy removal. The slit and eyepiece can readily be interchanged if great purity in the spectrum is desired; and, as just intimated, either one, two, or three half-prisms can be used, giving a dispersion or purity (as the case may be) of four, fifteen, or sixty ordinary prisms of 60°. The concave lens l has the effect of increasing the focal length of the collimator, and consequently of reducing the apparent breadth of the spectrum-lines for a given width of slit. At the same time the spectrum is made narrower and brighter; if, however, the spectroscope be applied to a large telescope, there is in fact no gain in light, a portion only of the pencil from the object-glass being used, whilst the width and height of the slit may be increased in the same proportion so as to keep the purity and breadth of spectrum constant. Notwithstanding this there is a practical advantage in the diminished effect of dust or irregularities on the slit. It remains to mention one important point in the direct-vision spectroscopes. Since the collimator and telescope are in a straight line, an image of the slit is formed in the centre of the field by rays which pass through the object-glasses above and below the prisms; and this serves as a bright reference line, which has the great advantage of being affected in the same way as the spectrum-lines by any shift of the collimator or viewing-telescope. Either a positive or negative eyepiece may be used with this ghost, and its brightness or colour may be varied by diaphragms or coloured glasses. Thus a convenient bright-line micrometer is formed, supplying a want which has long been felt by spectroscopists.

It may be remarked that this bright line gives a ready means of adjusting the collimator and telescope to focus, as, when this has been done, it should be seen distinctly at the same time as the spectrum-lines.

The success of this spectroscope has far exceeded all my anticipations. Though the first prism is only 0·75 inch in cross section, and the total length 11 inches, the power is far greater than that of any spectroscope with which I am acquainted, and this is accompanied by remarkable brightness in the spectrum. I have compared it carefully with the large spectroscope of the Royal Observatory, which has a train of ten compound prisms, and find that even with two prisms in the small direct-vision spectroscope the sodium lines are fully as widely separated as, and far better defined than, in the large spectroscope with its full power, whilst their brightness in the direct-vision spectroscope is incomparably greater. With the full train of three prisms this separation is quadrupled nearly, and the sodium lines are seen fully 3° apart with a power of 9 on the viewing-telescope. The compactness of the half-prism spectroscope, the simplicity of its construction, and the ease with which it is manipulated appear to leave little or nothing to be desired.

I have, however, tried a still more portable form of spectroscope, which for many purposes seems fully equal to the larger form. It is on the plan of the miniature spectroscopes, in which the eye takes the place of the viewing-telescope; and its general construction will be readily seen from fig. 2, which represents its actual size.

The prisms turn about centres *a*, *b*, being moved by the lever *d e f*, on which a micrometer-screw *m* acts as in the larger spectroscope; but to allow of more motion in the first half-prism the positions of the centres *a*, *b* have been altered, which can be done when there are only two prisms in the train, the condition between the lateral displacements still holding. The other arrangements are as before, the prism *b* being mounted in a small tube so as to be readily withdrawn if large dispersion is not required; but in place of the viewing-telescope and eyepiece, there is simply an eyeglass, *g*, adapted to the sight of the observer for distant objects, so that the rays pass through the prisms in a state of parallelism. The bright line formed by the image of the top and bottom of the slit is here seen above and below the spectrum, and measurements are made with great convenience.

As the whole of the magnifying-power has to be supplied by the prisms, I have increased the angle of the flint to 58° 30′, which approaches very closely to the critical angle; whilst in order to preserve the direct-vision form, the angle of the crown is 120°, giving a magnifying-power of 9 and a dispersion or purity (as the case may be) of 50°, or ten ordinary prisms of 60° for each half-prism. The train of two half-prisms has thus a magnifying-power of about 80, and a dispersion or purity of nearly 500°, or 100 ordinary prisms of 60°.

Small though this spectroscope is (the total length being only four

inches), its power is extraordinary. With the first half-prism alone the D lines are distinctly separated, whilst with the two half-prisms their angular distance is (by actual measurement) $\frac{2}{3}°$; and this separation, be it remembered, is obtained solely by the prisms, without any magnifying-power from an eyepiece. This separation is about equal to that in the large Greenwich spectroscope with its ten compound prisms of 1·6 inch section and the ordinary eyepiece, whilst the definition is fully as sharp and the brightness far superior. In fact where it is not necessary to magnify the breadth of the spectrum, the miniature spectroscope would appear to be quite equal to the ordinary form, as there is no use in increasing the height of the prisms beyond what is required to fill the pupil of the eye with light. Thus in the miniature spectroscope the prisms need be only 0·2 inch high; whilst if a magnifying-power of 10 is to be used (in the direction of the breadth of the spectrum) their height should be 2 inches to give the same brightness. Except in the case of stars and other objects of very limited apparent size, breadth may be given to the spectrum just as well by increasing the height of the slit as by increasing the height of the prisms. Of course the breadth of the first half-prism should be as large as may be, the magnifying-power in this direction being in all cases high enough to reduce the breadth of the pencil to a small fraction of the diameter of the pupil of the eye.

The results obtained when the train of prisms is reversed so as to give great purity are even more remarkable. With either of the half-prism trains in this position, without slit, collimator, or telescope, the sodium lines are seen with great brilliancy (though not divided) in the flame even of an ordinary paraffin or composite candle without any salt on the wick, and the Fraunhofer lines are seen sharply defined when the train of prisms pure and simple is pointed to the sun or moon. When the two trains are combined as in (5), page 29, still without any slit or lenses, the sodium-lines in an ordinary flame are widely separated, forming two images of the flame; and the *d* and *b* lines in the sun are also well divided. From the low altitude of the sun and the prevalence of haze, I have not had a fair opportunity of trying whether the prominences could be seen in this way without any slit or lenses; but from the ease with which the C line is seen, I think it probable that the prominences would be visible as a ring of light on the background of the spectrum formed by the successive coloured images of the sun's disk, dazzling though it is. Even if this plan should fail with these short trains of prisms, there would be no difficulty in adding one or two more prisms to each; for I imagine that the limit to the power is far from having been reached in the spectroscopes I have described. I have deferred making this new plan of spectroscope known until I could try it in a tolerably complete form, although the first experiments I made with direct-vision and other half-prisms in October last convinced me of its practical value. The results which have been obtained so far will, I hope, show that there is no difficulty in ap-

plying the method so as to obtain in a compact form and at a small cost a dispersive power exceeding that of any spectroscope on the old plan, and free from the defects inherent in a complicated instrument.

In conclusion, I may allude to an application of the remarkable property of the half-prism which may perhaps prove of practical use. From what precedes it will be clear that the half-prism, as far as its magnifying-power is concerned, is equivalent to a combination of an object-glass and cylindrical eyepiece, the peculiarity being that it magnifies the angle between two pencils of parallel rays without affecting the parallelism of the rays in each pencil. The half-prisms which have so far been considered are constructed so as to give great dispersion, and the sun is seen through them as if through a telescope with an object-glass formed of a convex flint lens and a concave crown. But it is obvious that an achromatic prism may be formed on the same principle as the achromatic object-glass, giving cylindrical magnifying-power without dispersion. If two such prisms be crossed at right angles, one behind the other, the magnifying-power will be the same in both directions, and the combination will act as an achromatic telescope with the advantage of great compactness, since the eye can be applied close to the second prism.

Whether, however, this advantage would compensate for the greater quantity of glass required and for the loss of light is a matter for practical consideration; and I therefore defer the discussion of the achromatic half-prism till I have satisfied myself that it has some practical utility, even though it may never replace the refracting telescope for general use.

Royal Observatory, Greenwich,
1877, Jan. 17.

March 8, 1877.

Dr. J. DALTON HOOKER, C.B., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

I. "On Magneto-electric Induction in Liquids and Gases.—Part I. Production of Induced Currents in Electrolytes." By J. A. Fleming, B.Sc. (Lond.). Communicated by Prof. Stokes, Sec.R.S. Received February 6, 1877.

(Abstract.)

This paper contains an account of an experimental inquiry into the production of induced currents in liquids by magneto-electric induction. Faraday examined one such case of induction, in which a conducting

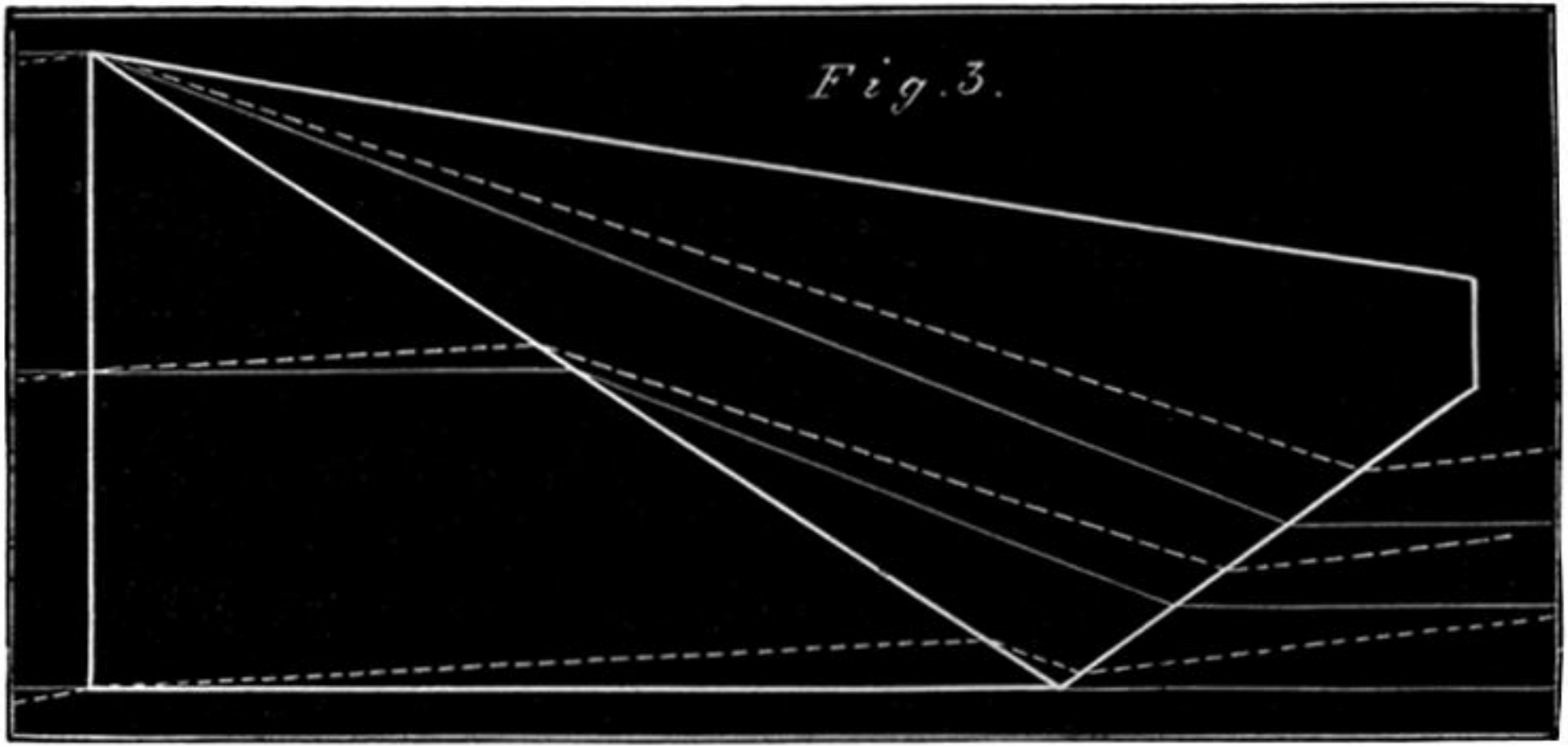
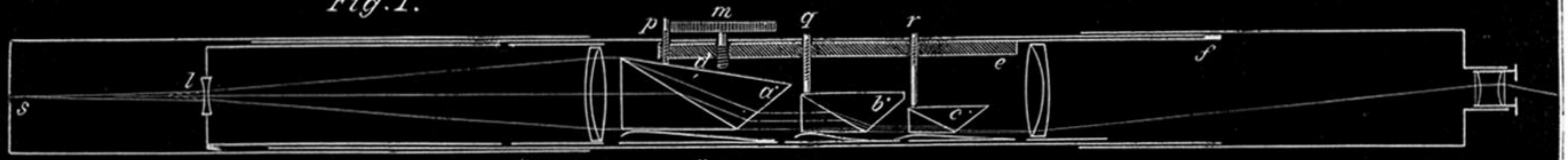


Fig. 3.

Fig.1.



"Half-prism" Spectroscope

Fig.1.a

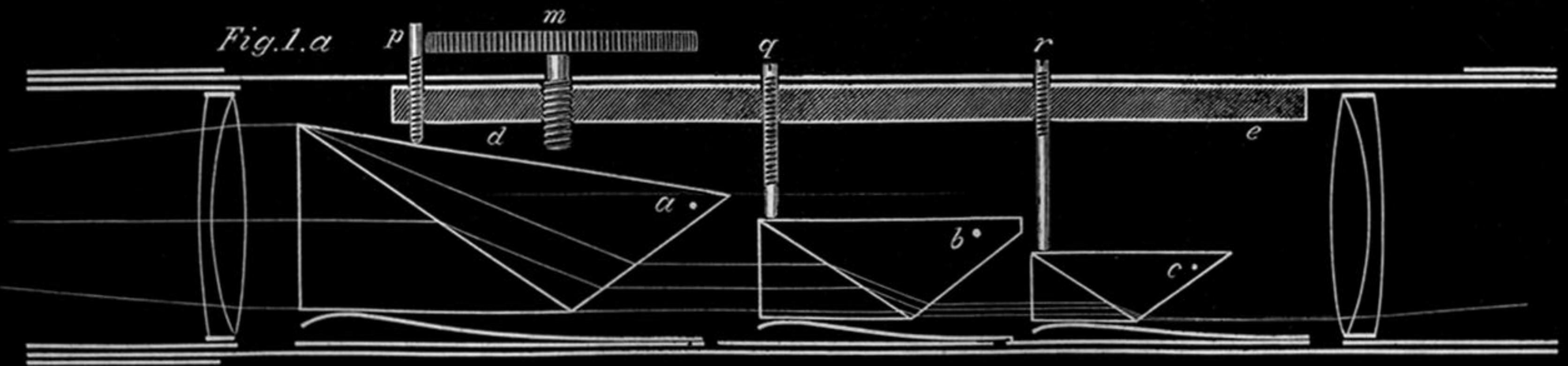


Fig.2. Miniature "Half-prism" Spectroscope.